2.5 Continuity

2014-09-19 day 18

12a) the Earth's pop as a f^n of time
NOT continuous $\lim_{t \to now} P(t)$ DISCRETE

b) your exact height as a f^n of time
continuous

c) the cost of a taxi ride in your "city" as a f^n of distance travelled
discrete discreet

d) Volume of a melting ice cube as a f^n of time
depends on "granularity"

Progression of Ideas

→ define "continuous at a point"
1 →
2 →
3 →
limit
ε-δ definition
more precise
I don't care if f is defined at a.

→ define "continuous on an open interval"
* continuous at every pt in the interval.

→ define "continuous at an endpoint of an interval"
* use right-handed or left-handed one sided limits in place of two sided limits above

"continuous in 3 dimensions"
* f(x,y)

$A(x,y)=x+y$

$S(x,y)=x-y$

2.5 Continuity

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21) $|x^2-2x|$

"Polynomials are continuous everywhere"

"Absolute value of continuous fn is continuous"

2.5 Continuity

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18) $f(x) = \dfrac{3x+1}{x^2+7x-2}$

where is this not defined?

$x = \dfrac{-7 \pm \sqrt{49-4(1)(-2)}}{2}$

f(x) is a rational f^n, so

"Rational f^n's are continuous everywhere they are defined"

⇒ this (f(x)) cont. everywhere except

2.5 Continuity

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2.6 Limits and continuity of trig f^n's

→ squeeze theorem interesting idea
not major milestone

- major memorization test

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Corollary

$$0 = \lim_{x \to 0} \frac{1-\cos x}{x} = \lim_{x \to 0} \frac{\cos x - 1}{x}$$

- very interesting example at the bottom of pg 162.

2.5 Continuity

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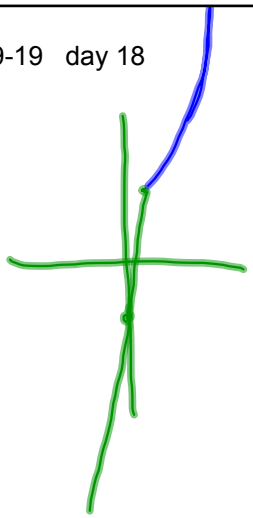
2.5/25

a) $f(x) = \begin{cases} 7x-2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$



→ f(x) def. @ 1

→ $\lim_{x \to 1} f(x)$ exist?

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 7x-2$

$= 7-2 = 5$ ♡

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} kx^2 = k(1)^2$

$= k.$

Does the 2sided limit exist?

only when $5=k$

III if $k=5$, then

$f(1) = 5 = \lim_{x \to 1} f(x)$ ✓

$f(1) = 7(1)-2$

$f(1) = 5$ $\quad 5 = k(1)^2$

$5 = k$

2.5 Continuity

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25b) $f(x)=\begin{cases} kx^2, & x \le 2 \\ 2x+k, & x>2 \end{cases}$

1) $f(2)$ ✓

2) $\lim_{x\to 2^-} kx^2 = 4k$

$\lim_{x\to 2^+} 2x+k = 4+k$

exactly the same when

$4k = 4+k$

$-k \quad -k$

$3k = 4$

$\frac{3k}{3} = \frac{4}{3}$

$k = \frac{4}{3}$

3) $f(2) = 4k = \left(\frac{4}{3}\right)(4) = \frac{16}{3}$

$= \lim_{x\to 2} f(x) = 4+\frac{4}{3} = \frac{12+4}{3} = \frac{16}{3}$ ✓

2.5 Continuity

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24) $f(x) = \begin{cases} \frac{3}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$

where not cont?

continuous everywhere except (possibly) at 1

cont everywhere except perhaps where the glue is (at $x=1$)

1) $f(1) = 3$

2) $\lim_{x \to 1} \frac{3}{x-1} = \begin{cases} +\infty \\ -\infty \\ \text{DNE} \end{cases}$ limit doesn't exist

Not continuous.

"$\frac{3}{0ish}$" tells me limit does not exist and I would write $+\infty$ or $-\infty$ or DNE